The Physics of the Sky Crane
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1 Introduction

This year's Botball game involved the introduction of the "sky crane", and can be seen in the video provided in this link: http://youtu.be/wg8KmOUUCtI[1]. For the purposes of this paper, the directions "forward", "backward", and "vertical" will be defined as the movement of the sky crane in a direction parallel to the black line (seen 0:15 to 0:24). The directions "left", "right", and "horizontal" are defined as the movement of the sky crane in a direction perpendicular to the black line (seen 0:05 to 0:14). As the video demonstrates, the sky crane exhibits rotational motion in the vertical direction, and lateral motion in the horizontal direction.

The sky crane proved to be a formidable obstacle in the regional competitions, stopping many team's robots from passing through and obtaining Botguy or the pom-poms. The successful teams usually drove around or under the sky crane, and were limited in their design space by the very small maneuvering room they had. Larger, more complicated robots, most notably ones using an iRobot Create base, often could not traverse the sky crane and ended up foundering, unable to score points for their respective team. Our team, from Winchester High School, was able to pass through the sky crane by utilizing its ability to move laterally in the horizontal direction, pushing it to the side, as demonstrated in this video: http://youtu.be/UC2ekhfhcGg[2]. However, the sky crane itself presents an interesting problem: attempting to push from the bottom of the crane, as demonstrated in ***Figure 1.1***, causes it to lock up and it will not move. Instead, the robot must push in a different location, as demonstrated in ***Figure 1.2***, in order to move the crane. This paper seeks to explain this phenomenon.

1.1 Sky Crane Pictures

Figure 1.1

Figure 1.2

2 Purpose

Moving the sky crane horizontally provides many distinct advantages, as opposed to driving around or through it. By moving the sky crane to the side of the game board, it is neutralized for the remainder of the round, allowing robots to pass back and forth, opening up many more options for game strategies. Additionally, by moving the sky crane to the side, more complex structures can be built and mounted onto robots, as there is more room for maneuvering and less chance of obstruction and subsequent entanglement and/or breakage.

More importantly, by moving the sky crane to the side, Creates, which normally cannot fit under the sky crane, can pass through and reach the pom-poms, Botguy, and the other side of the board from both directions. Due to the fact that the Create is very powerful, fast, and precise compared to the Lego robot, it is much more versatile and can be used for both fast, efficient point gathering, and for disruption, an integral strategy for double elimination. Without the ability to quickly and safely neutralize the sky crane, such disruption strategies are completely unfeasible.

3 Mathematics

3.1 Force Diagrams

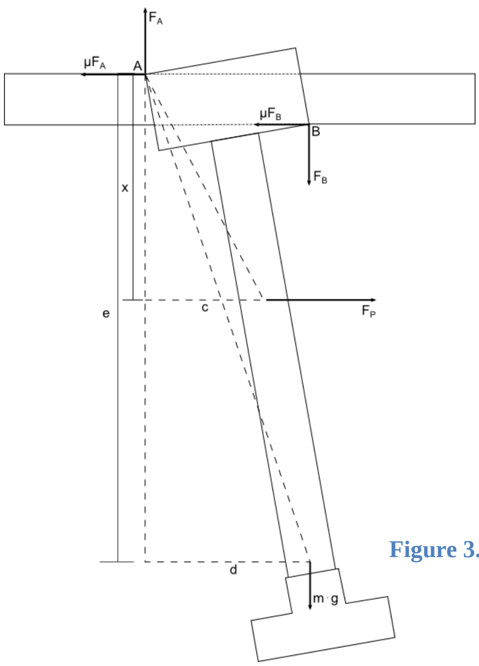

Figure 3.1

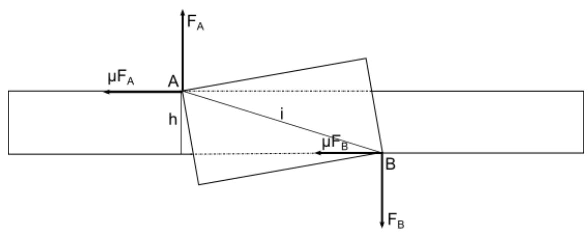

Figure 3.2

3.2 Constants Explained

When the sky crane is pushed horizontally, the top T-connector makes contact with the PVC at two locations, as seen in ***Figure 3.2***, which is a detailed diagram of the top of ***Figure 3.1***. The upper and lower points are denoted A and B, respectively. At each of these points a force is exerted by the PVC on the crane, denoted by F_A and F_B, respectively. The constant μ is the coefficient of static friction between the PVC. The variable F_p represents the force of the push by the robot on the crane. The constant m is the mass of the entire sky crane and g represents the acceleration due to gravity. The force mg originates from the sky crane's center of mass. All other constants (c, d, e, h, i, x) represent distances between points.

3.3 Deriving Equations

Our goal is to find F_p as a function of x. In other words, we want to find the minimum amount of force required to get the sky crane moving at a given, but unknown velocity. We start by finding the net torque using point A as the axis of rotation:

$$\sum \tau = \tau_p - \tau_{mg} - \tau_{\mu F_b} - \tau_{F_b}$$

$$\sum \tau = F_p \cdot \sqrt{x^2 + c^2} \cdot \sin(\tan^{-1}\frac{x}{c}) - mg \cdot \sqrt{e^2 + d^2} \cdot \sin(\tan^{-1}\frac{e}{d}) - i \cdot \mu F_b \cdot \sin(\sin^{-1}\frac{h}{i}) - i \cdot F_b \cdot \sin(\frac{\pi}{2} + \sin^{-1}\frac{h}{i})$$

This simplifies to:

$$\sum \tau = F_p \cdot x - mg \cdot e - \mu F_b \cdot h - F_b \cdot \sqrt{i^2 - h^2}$$

We also need the net force on the object in the horizontal and vertical directions:

$$\sum F_{vertical} = F_A - F_b - mg \qquad \sum F_{horizontal} = F_p - \mu \cdot F_b - \mu \cdot F_A$$

Since the crane is in static equilibrium, net torque and net force in both directions equal 0. This gives us a system of three equations:

$$F_p \cdot x = mg \cdot e + \mu F_b \cdot h + F_b \cdot \sqrt{i^2 - h^2} \tag{1}$$

$$F_A = F_b + mg \tag{2}$$

$$\frac{F_p}{\mu} = F_b + F_A \tag{3}$$

Substituting Equation 2 into Equation 3 we get:

$$F_b = \frac{\frac{F_p}{\mu} - mg}{2} \tag{4}$$

Substituting Equation 4 into Equation 1 we get:

$$F_p \cdot x = mg \cdot e + \mu \cdot \frac{\frac{F_p}{\mu} - mg}{2} \cdot h + \frac{\frac{F_p}{\mu} - mg}{2} \cdot \sqrt{i^2 - h^2}$$

This simplifies to our final equation:

$$F_p = \frac{mg\left(2e - h\mu - \sqrt{i^2 - h^2}\right)}{2x - h - \frac{\sqrt{i^2 - h^2}}{\mu}}$$

3.3 Finding Constants

The mass of the moving part of the crane (m) was determined with an electronic balance. The constants i and h were found by measuring the dimensions of the T-connector with a measuring tape. Below is a table of the constants and their respective values:

Table 3.3

Constant	Value
m	$0.992\ kg$
g	$9.8\ m/s^2$
h	$0.032\ m$
i	$0.113\ m$

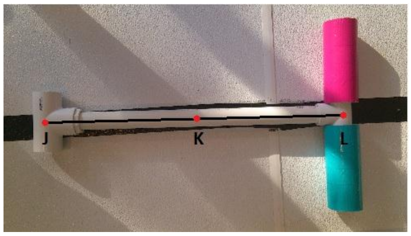

Figure 3.4

3.3.1 Finding *e*

To find e we must first find the center of mass of the sky crane. We can treat the sky crane as three points of mass, as seen in ***Figure 3.4***. Point J is the center of mass of the PVC T-connector at the top, Point K is the center of mass of the central PVC pipe, and Point L is the center of mass of the boosters, bottom T-connector, and PVC pipes. The center of mass is calculated by the formula:

$$r_{cm} = \frac{m_k \cdot r_1 + m_L \cdot (r_1 + r_2)}{m}$$

Using $0.2345\ kg$ for m_k, $0.5395\ kg$ for m_L, $0.28\ m$ for r_1, $0.27\ m$ for r_2, and the equation above, we get $0.393\ m$ for r_{cm}. After marking the point found on the sky crane, the measured value of e was found to be $0.416\ m$.

3.3.2 Finding *μ*

We chose to find the coefficient of static friction (μ) experimentally. To find μ, the sky crane was placed in a position at rest, and one end was elevated until the arm began to slide down the PVC. The height of elevation was recorded as y. Knowing that the length of the horizontal pipe is $0.6096\ m$, we can find μ using the following relation, with θ being the angle of elevation:

$$\mu = \tan\theta$$ [3]

Because $\theta = \sin^{-1}\frac{y}{0.6096}$, this simplifies to:

$$\mu = \frac{y}{\sqrt{0.6096^2 - y^2}}$$

Below are ten trials we took:

Table 3.5

Height (y)	Coefficient of static friction (μ)	Height (y)	Coefficient of static friction (μ)
0.0762 m	0.126	0.0889 m	0.147
0.0810 m	0.134	0.0905 m	0.150
0.0921 m	0.153	0.1016 m	0.169
0.0953 m	0.158	0.0968 m	0.161
0.0857 m	0.142	0.1016 m	0.169

Using the data collected, we took a t-interval sample with a 95% confidence rate to find an acceptable interval of values for μ[4]. The interval was calculated to be $(0.1407, 0.1611)$, and the average μ value was found to be 0.1509.

3.4 Graph

Having found all the constants, we can plug them into the equation we derived in ***Section 3.3***, giving us the equation:

$$F_p = \frac{3.36}{x - 0.377}$$

When we graph this equation with x $(meters)$ on the x-axis and F_p $(newtons)$ on the y-axis, we get:

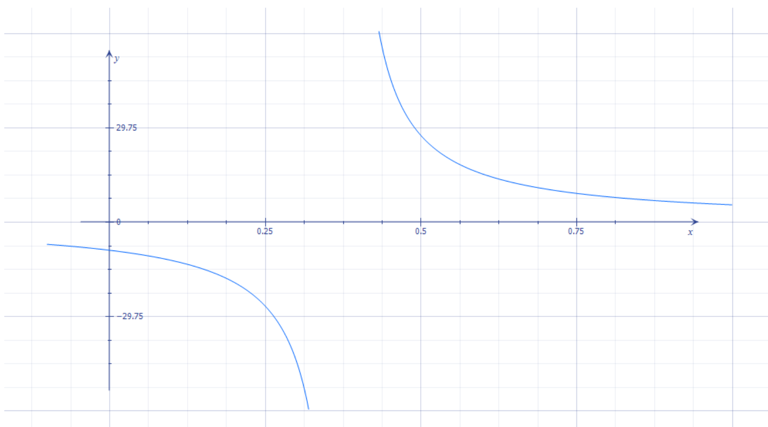

Figure 3.6

4 Analysis

From the graph above we can see that there is an asymptote around 40 cm. More specifically, we know it is approximately 37.7 cm since that is the x-shift in our equation.

To the right of the asymptote, the amount of force required to push the sky crane is astronomical. Although the amount of force required decreases as x increases, because the length of the arm of the sky crane is 55 cm, the minimum force needed to move the sky crane between 37.7 cm and 55 cm is 19.42 N, which is still out of the ability of the Create.

To the left of the asymptote, the function breaks down. Normally, it does not make sense for the force to be negative, since then all torques would be in the same direction, and would not cancel out. However, it does mean that the situation shown in ***Figure 1.1*** will never happen, no matter how much force is exerted, due to the fact that the torque of F_p is small enough that the torque of mg cancels it out and prevents the crane from tilting, or that F_p is strong enough to overpower the force of friction. This means that any force applied above the 37.7 cm mark on the sky crane will be able to move the sky crane to the side.

5 Conclusion

Overall, we have discovered that the act of moving the sky crane to a side is not an impossible task, as it can be done with little effort with a structure that pushes the crane at a distance less than 37 cm from the top, which is well under the 15 inch height limit of the starting box. However, if the robot is pushing at an angle, creating greater torque and subsequently greater static friction, or if the robot is pushing too low, the crane will be completely immovable. As a result, we suggest that any team wishing to move the sky crane to push from a point of contact as high as possible.

If you wish to contact us about any questions you may have, please email us at whsbotball@gmail.com.

6 References

[1] "Sky Crane." *YouTube*. YouTube, 17 Jan. 2013. Web. 31 May 2013. <http://www.youtube.com/watch?v=wg8KmOUUCtI>

[2] "Winchester 2013 Regionals." *YouTube*. YouTube, 05 May 2013. Web. 31 May 2013. <http://www.youtube.com/watch?v=UC2ekhfhcGg>.

[3] Boston University, 27 July 2000. Web. 31 May 2013. <http://physics.bu.edu/~duffy/semester1/c6_measuremus.html>.

[4] "Student's T-distribution." *Wikipedia*. Wikimedia Foundation, 31 May 2013. Web. 31 May 2013. <http://en.wikipedia.org/wiki/Student's_t-distribution>.